DRAFT

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History:

When I saw sequences A324337 (= A002487∘A006068) and A324338∘(= A002487∘(1+A006068)) from Antti Karttunen, Feb 23 2019, I added in OEIS a comment to both sequences (in addition to other properties):

For all n > 0 A324338(n)/A324337(n) constitutes an enumeration system of all positive rationals. For all n > 0 A324338(n) + A324337(n) = A071585(n). - Yosu Yurramendi, Oct 22 2019

On November 6th I wrote him by personal mail to communicate my comment. On the same day he answered me with this question:

'**It would be interesting to know what kind of family of bijections p on N allow that A002487(1+p(n))/ A002487(p(n)) constitute an enumeration system for positive rationals?**'

On November 8th I partially answered him the question:

'At this moment I can't answer you precisely your question, but I found that all the permutations en my OEIS webpage have this property. This fact is very interesting for me (thanks), because I could generate a lot of enumeration systems based on A002487. Some of them find again systems already defined. I found also a curious fact with one of them, may be you know it (it's you who defined sequence A233279):

A324338 = A002487[1+A006068] = A002487[A233279]

A324337 = A002487[A006068] = A002487[1+A233279]'.

On December 31th, I followed by answering:

'I have a conjecture about your question above. 'If p is a bijection on N such that it permutes by levels [2^m, 2^(m+1)), m>=0, then A002487(1+p(n))/ A002487(p(n)) constitutes an enumeration system for positive rationals.' I tried out several times with the following random R-program, and I verified the conjecture: (p <- c(1, sample(2:3), sample(4:7), sample(8:15), sample(16:31), sample(32:63), sample(64:127)))'

I little bit later:

'I have to precise a little bit the conjecture. In fact, (p <- c(sample(1:127))) gives also the beginning of a positive rational

enumeration system, but all the fractions of 0<=m<=6 levels are mixed. It doesn't preserve at each level those of A002487 (numerator and denominator). So, I have to add to the conjecture the constraint that at each level the numerator and the denominator are the same as those of A002487.'

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R-program (https://www.r-project.org/)

A002487[1] = 1

A002487[2*n] = A002487[n]

A002487[2*n+1] = A002487[n] + A002487[n+1]

∀m ≥ 0   A002487(2^m) = 1 (proof by induction)

```
> A002487[1:31] ; A002487[1+1:31] # A002487[1:31] / A002487[1+1:31] is the beginning of an enumeration system
 [1]  1    1  2    1  3  2  3    1  4  3  5  2  5  3  4    1  5  4  7  3  8  5  7  2  7  5  8  3  7  4  5
 [1]  1    2  1    3  2  3  1    4  3  5  2  5  3  4  1    5  4  7  3  8  5  7  2  7  5  8  3  7  4  5  1
```

A002487[1+n] has the same terms as A002487[n] at each block m ≥ 0, 2^m ≤ n < 2^(m+1).

In effect, A002487[2^m+1], A002487[2^m+2, ..., A002487[2^(m+1)-1] are common to both A002487[n] and A002487[1+n], and A002487[2^(m+1)] = A002487[2^m].

See for m = 3, A002487[16] = A002487[8]

A random permutation 'sigma' by blocks:

```
> (sigma <- c(1, sample(2:3,2), sample(4:7,4), sample(8:15,8), sample(16:31,16)))
 [1]  1    2  3    4  6  7  5    15 14 10  9  8 11 12 13    21 28 26 25 27 16 17 19 31 24 22 30 23 29 18 20
> 1+sigma
 [1]  2    3  4    5  7  8  6    16 15 11 10  9 12 13 14    22 29 27 26 28 17 18 20 32 25 23 31 24 30 19 21
> A002487[sigma] ; A002487[1+sigma]
 [1]  1    1  2    1  2  3  3    4  3  3  4  1  5  2  5    8  3  5  7  8  1  5  7  5  2  5  4  7  7  4  3
 [1]  1    2  1    3  3  1  2    1  4  5  3  4  2  5  3    5  7  8  5  3  5  4  3  1  7  7  5  2  4  7  8
```

See A002487[16] = A002487[8]

It's sure (A002487(2^m) = 1) that at each block all the terms of A002487 will be over there.

So, ∀m ≥ 0, 2^m ≤ n < 2^(m+1)
A002487[sigma(n)]/A002487[1+sigma(n)] are all irreducible fractions belonging to block m, such as A002487[n]/A002487[1+n], but, in general, in a different order, by following sigma.